

LINKING CHIRAL AND TOPOLOGICAL ASPECTS OF THE QUARK-QUARK INTERACTION, IN QCD*

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We present the results of a recent investigation of the short-distance structure of non-perturbative interaction between light quarks. We analyze the connection between topology and the dynamics responsible for the spontaneous breaking of chiral symmetry. By measuring some appropriate correlation functions on the lattice, we provide compelling evidence for instanton-induced dynamics.

1. Introduction

Understanding the light-quark dynamics at all scales is a fundamental and open problem, in high-energy physics. In particular, the improving our knowledge of the non-perturbative interaction at intermediate and short distances has become essential, in order to explain the results of recent experiments on pion and proton electro-magnetic formfactors, performed at Jefferson Laboratory ¹. Indeed, these high-precision measurements have shown that the perturbative regime in QCD does not set-in until very large values of the momentum transfer ($Q^2 > 6 \text{ GeV}^2$). The delay of the onset of the asymptotic regime implies that there are strong, short-scale non-perturbative interactions, inside ordinary hadrons.

It is commonly accepted that the physics of the light-quark sector of QCD is strongly influenced by the dynamics responsible for the chiral symmetry breaking (CSB). The characteristic energy scale associated to such

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a non-perturbative phenomenon is $4\pi f_\pi \sim 1.2 \text{ GeV}$, sizably larger than the typical confinement scale, Λ_{QCD} . Such a separation of scales justifies attempting to understand the short-distance non-perturbative structure of light-hadrons, without having simultaneously to understand microscopic origin of color confinement. On the other hand, from the observation that $4\pi f_\pi \sim m_{\eta'}$ it follows that topological effects should be included in any effective description of the light-quark dynamics.

An effective interaction^a, which can simultaneously solve the U(1) problem and explain the CSB was derived by 't Hooft², by computing the Euclidean QCD generating functional in the semi-classical limit (i.e. accounting for small perturbations around the instanton solution):

$$L'_{tH} = G_{\bar{\rho}, \bar{n}} \left([(\psi^\dagger \tau_a^- \psi)^2 - (\psi^\dagger i\gamma_5 \tau^- \psi)^2] + \frac{1}{2(2N_c-1)} (\psi^\dagger \sigma_{\mu\nu} \tau_a^- \psi) \right),$$

where $\tau^- = (\vec{\tau}, i)$ ($\vec{\tau}$ are isospin Pauli matrices), and $G_{\bar{n}\bar{\rho}}$ is a coupling constant depending on the typical density of instantons in the vacuum \bar{n} , and their typical size $\bar{\rho}$. Notice that the finite size of the instanton field provides a natural cut-off scale for the interaction.

Unfortunately, the instanton density and size cannot be computed systematically in QCD, so the semi-classical 't Hooft interaction is not completely specified. The Instanton Liquid Model (ILM)³ consists of assuming that the vacuum is saturated by instantons with typical size $\bar{\rho} \simeq 1/3 \text{ fm}$ and density $\bar{n} \simeq 1 \text{ fm}^{-4}$.

In a number of recent works it was shown that instantons can quantitatively explain the observed deviation of the pion and nucleon electromagnetic formfactor from the perturbative prediction, at high momentum transfer⁴. In the present work, we complement that analysis, and use lattice simulations to look directly for the specific signatures of the 't Hooft interaction.

2. Instantons and Chirality-Mixing Interactions

A characteristic property of the 't Hooft effective vertex is that it mixes only quarks of *different chirality*. Hence, if instantons are the dominant sources of the non-perturbative interaction, then one should be able to observe that quarks change their chirality after each non-perturbative interaction. On the contrary, if the non-perturbative vertex has the same structure of the perturbative gluon exchange (i.e. a purely vector or axial-vector coupling), then a single interaction should not flip the chirality of quarks. In this case,

^aFor simplicity, in this work we shall restrict to the $N_f = 2$ case.

chirality mixing should arise only as a consequence of mass generation, i.e. from a genuine many-body effect.

Based on this observations, one is led to consider the ratio:

$$R^{NS}(\tau) := \frac{A_{flip}^{NS}(\tau)}{A_{non-flip}^{NS}(\tau)} = \frac{\Pi_{\pi}(\tau) - \Pi_{\delta}(\tau)}{\Pi_{\pi}(\tau) + \Pi_{\delta}(\tau)}, \quad (1)$$

where $\Pi_{\pi}(\tau)$ and $\Pi_{\delta}(\tau)$ are pseudoscalar (PS) and scalar (S) iso-triplet two-point correlators, related to the currents $J_{\pi}(\tau) := \bar{u}(\tau) i\gamma_5 d(\tau)$ and $J_{\delta}(\tau) := \bar{u}(\tau) d(\tau)$. If the propagation is chosen along the time direction, $A_{flip(non-flip)}^{NS}(\tau)$ represents the probability amplitude for a $|q\bar{q}\rangle$ pair with isospin 1 to be found after a “time” interval τ in a state in which the chirality of the quark and anti-quark is interchanged (not interchanged). Notice that the ratio $R^{NS}(\tau)$ must vanish as $\tau \rightarrow 0$ (no chirality flips), and must approach 1 as $\tau \rightarrow \infty$ (infinitely many chirality flips).

It can be shown that the correlator (1) receives no perturbative contribution. Moreover, a model-independent spectral analysis indicated that the interaction responsible for dynamical chirality-mixing is mediated by topological fields, and that the rate of chirality flips in a quark-antiquark system is proportional to the mass of the η' meson⁵.

The chirality flip ratio $R^{NS}(\tau)$ was recently investigated in QCD, by means of lattice simulations performed with chiral fermions⁶. The results are the square points plotted in Fig. 1(A). As expected, the lattice data interpolate between 0 and 1. We observe that after few fractions of a fermi the quarks are more likely to be found in a configuration in which their chiralities is flipped, than to be found in their initial configuration. The presence of a maximum in $R^{NS}(\tau)$, and the subsequent fall-off toward 1 have a very simple explanation in the ILM: if quarks propagate in the vacuum for a time comparable with the typical distance between two neighbor instantons (i.e. two consecutive 't Hooft interactions), they have a large probability of crossing the field of the closest pseudo-particle. If so happens, they must necessarily flip their chirality. So, after some time, the quarks are most likely to be found in the configuration in which their chirality is flipped. On the other hand, if one waits for a time much longer than 1 fm, then the quarks will “bump” into many other such pseudo-particles, experiencing several more chirality flips. Eventually, either chirality configurations will become equally probable and $R^{NS}(\tau)$ will approach 1.

Let us now consider the result for $R^{NS}(\tau)$ obtained in the Random

Instanton Liquid Model^b (circles in Fig. 1(A)). The agreement with the lattice results is impressive, implying not only that instantons are the relevant sources of short-range chirality-mixing interaction, but also that the phenomenological estimates $\bar{\rho} \simeq 1/3 \text{ fm}$, $\bar{n} \simeq 1 \text{ fm}^{-4}$ are realistic.

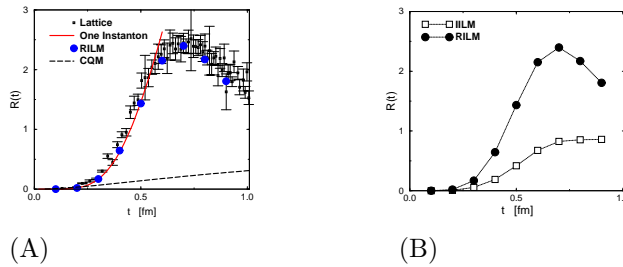


Figure 1. (A): The chirality-flip correlator in lattice QCD (square points) and in the RILM (circles). The solid line represent the single-instanton contribution. The dashed curve was obtained from two free “constituent” quarks with a mass of 400 MeV. (B): Suppression of chirality flipping events, due to fermion-loop exchange in the ILM. Circles are RILM (quenched) results, squares are ILM (unquenched) results.

3. Instanton, Unitarity and Quark Loops

In this section we discuss an interesting link between topology and unitarity, which emerges when one analyzes the contribution of quark loops to chirality flips. We recall that lattice results reported in section 2 have been obtained in the quenched approximation. Unitarity requires that all correlation functions must be positive definite and this implies that $R^{NS}(\tau) \leq 1$, at all times. The existence of a maximum in which the chirality flip ratio is greater than one is a reflection of the fact that, in the quenched approximation, the unitarity of the theory is lost. We necessarily conclude that the fermionic determinant suppresses some chirality-flipping events, which are otherwise allowed in the quenched approximation. Clearly, *realistic descriptions of the effective quark-quark interaction must reproduce a dramatic enhancement of the chirality flipping amplitude, when quark loops are suppressed.*

^bIn such a version of the ILM quarks are assumed to propagate in a vacuum populated by an ensemble of randomly distributed instanton and anti-instantons. It can be shown that the RILM accounts for the 't Hooft interaction to all orders, but neglects quark loops (quenched approximation).

Quite remarkably, this phenomenon is naturally explained in the instanton picture. If quark loops are allowed, then instantons and anti-instantons can interact through fermion exchange. Such an interaction generates an attraction between instantons and anti-instantons leading to a screening of the topological charge. As a result of such a screening, quarks crossing the field of an instanton are very likely to find, in the immediate vicinity, an anti-instanton which restores their initial chirality configuration. In Fig. 1(B) we compare the chirality flipping ratio $R^{NS}(\tau)$ obtained from a quenched (RILM) and unquenched (IILM^c) version of the ILM. We observe that, with the inclusion of the fermionic determinant, the unitarity condition $R^{NS}(\tau) < 1$ is restored. We stress that, although such a restoration must necessarily take place in QCD, it represents a remarkable success of the ILM, which is not a unitary field theory.

4. Conclusions

We have presented a study of the quark dynamics, at intermediate and short distances, based on the results of lattice QCD simulations with chiral fermions. Our analysis provides strong evidence that instantons represent the main source of non-perturbative interaction between light quarks. We studied the contribution of quark loops, by comparing the results of quenched and full simulations. We observed dramatic quenching effects that can be naturally explained by instantons and suggests an interesting link between chiral dynamics, unitarity and topology.

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^cInteracting Instanton Liquid Model